# A scheme for measuring vibrational frequency and coupling strength in a coupled nanomechanical resonator-quantum dot system

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# A scheme for measuring vibrational frequency and coupling strength in a coupled nanomechanical resonator-quantum dot system

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The authors theoretically propose a precise way to measure the vibrational frequency and coupling strength in a coupled nanomechanical resonator-quantum dot system in terms of mechanically induced coherent population oscillation. The pump-probe spectroscopy that exhibits new features such as mechanically induced three-photon resonance and ac Stark effect is obtained. Simultaneously, the coupling strength between nanomechanical resonator and quantum dot can also be detected from Rabi-splitting-like peak in the probe spectrum. Our results could be applied to other systems such as coupled DNA-quantum dot systems. © *2009 American Institute of Physics*. [DOI: 10.1063/1.3072599]

Nanomechanical systems are drawing interest from both technical and scientific communities in physics, biology, medicine, and chemistry.[1–5] Because of the limited environment and small size, they can be used in the nano-/micron electronic and mechanical systems.[6,7] Another exciting possibility is to *physically couple* nanomechanical resonator to a condensed matter system such as a semiconductor quantum dot. Such a coupled quantum system can be used to study fundamental quantum effects, and also it has possible applications in high precision measurement,[8] zeptogram-scale mass sensing,[9] and laser cooling of a nanomechanical resonator mode to its quantum ground state.[10] The direct resonant interaction between atoms and a nanomechanical resonator has been investigated by Wang *et al.*[8] Yang *et al.*[9] demonstrated that nanomechanical systems can ultimately provide inertial mass sensing of individual intact electrically neutral macromolecules with single-Dalton resolution. In addition, Treutlein *et al.*[11] showed that a Bose–Einstein condensate (BEC) coupled to a nanomechanical resonator on an atom chip can be used as a sensitive quantum probe. All of these applications apparently require a suitable frequency for their appropriate processing. However, there is an incomplete technology for detecting the frequency of nanomechanical resonator with mechanical methods. For most nanomechanical systems, especially in bilayer or multilayer structures, the internal strains must be taken into account when estimating resonance frequencies,[1] which will cause the errors of the frequency measurement.

Recently, Roukes and co-workers[12–14] used the balanced magnetomotive detection and the balanced electronic detection technique to measure the fundamental resonance frequencies of nanomechanical systems. However, an accurate and effective method to measure the vibrational frequency of a nanomechanical resonator is still lacking. In the present letter, we propose a realistic, feasible, and exact way to measure the vibrational frequency of nanomechanical resonator with the optical spectrum. Theoretical analysis shows that two sharp peaks appeared in the probe absorption spectrum, which correspond to the frequency of nanomechanical resonator. The coupling strength between quantum dot and nanomechanical resonator can also be measured by the distance of the "Rabi-splitting-like" peaks.

We consider a system composed of a semiconductor quantum dot and a nanomechanical resonator in the simultaneous presence of a strong pump field and a weak probe field. The physical situation is illustrated in Fig. 1. At low temperatures, the semiconductor quantum dot can be modeled as a two-level system, which consists of the ground state $|0\rangle$ and the first excited state (single exciton) $|\mathrm{ex}\rangle$.[15,16] The atomic two-level system driven by a continuous bichromatic field was already studied by Yu *et al.*[17] As usual, the two-level exciton can be characterized by the pseudospin $-1/2$ operators $S^{\pm}$ and $S^{z}$. Then the Hamiltonian of exciton in a quantum dot can be described by $H_{\mathrm{QD}}=\hbar\omega_{\mathrm{ex}}S^{z}$, where $\omega_{\mathrm{ex}}$ is the exciton frequency. In a structure where the thickness of the beam is smaller than its width, the lowest-energy resonance corresponds to the fundamental flexural mode that will constitute the resonator mode.[10] Then the Hamiltonian of this resonator mode is given by $H_n=\hbar\omega_n a^{+}a$, where $\omega_n$ is the frequency of the resonator mode and $a$ ($a^{+}$) is the annihilation (creation) operator for the resonator mode. Since the

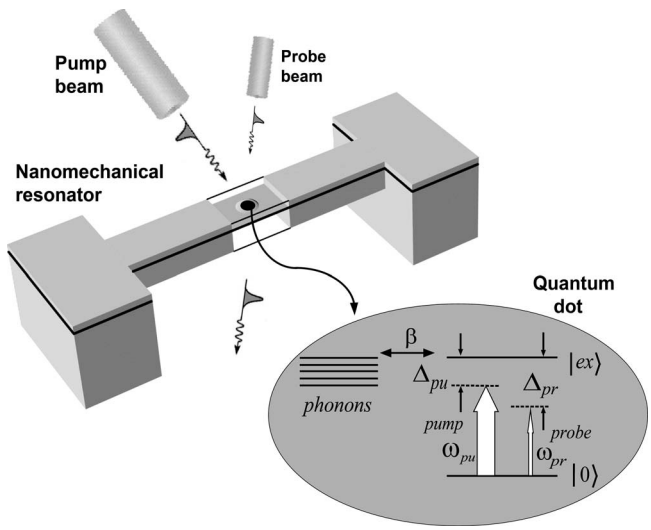


FIG. 1. Schematic diagram of a GaAs nanomechanical resonator with an embedded InAs quantum dot in the presence of a strong pump field and a weak probe field. The inset is an energy-level diagram of a quantum dot.

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flexion induces extensions and compressions in the structure of Fig. 1,[18] this longitudinal strain will modify the energy of the electronic states of quantum dot through deformation potential coupling. Then the Hamiltonian of the resonator mode coupled to the quantum dot is described by[10] $H_{\mathrm{QD}-n}=\hbar\omega_n\beta S^z(a^+ + a)$,where $\beta$ is the coupling strength of the resonator mode-quantum dot. As a strong pump field and a weak probe field are simultaneously applied to this coupled quantum dot-nanomechanical resonator system, the quantum dot via exciton interacts with them. We treat these optical fields classically.[19] Recently, such a coherent optical spectroscopy of a strongly driven quantum dot without a nanomechanical resonator was investigated experimentally by Xu *et al.*[20,21] The Hamiltonian of the quantum dot coupled to the pump field and probe field is as follows:[19] $H_{\mathrm{QD}-p}=-\mu(S^+E_{\mathrm{pu}}e^{-i\omega_{\mathrm{pu}}t}+S^-E^*_{\mathrm{pu}}e^{i\omega_{\mathrm{pu}}t})-\mu(S^+E_{\mathrm{pr}}e^{-i\omega_{\mathrm{pr}}t}+S^-E^*_{\mathrm{pr}}e^{i\omega_{\mathrm{pr}}t})$, where $\mu$ is the electric dipole moment of the exciton, $\omega_{\mathrm{pu}}$ ($\omega_{\mathrm{pr}}$) is the frequency of the pump field (probe field), and $E_{\mathrm{pu}}$ ($E_{\mathrm{pr}}$) is the slowly varying envelope of the pump field (probe field). Therefore, we obtain the total Hamiltonian of the coupled quantum dot-nanomechanical resonator in the presence of two optical fields,[10,19]

$$H = H_{\mathrm{QD}} + H_n + H_{\mathrm{QD}-n} + H_{\mathrm{QD}-p} = \hbar\omega_{\mathrm{ex}}S^z + \hbar\omega_n a^+ a + \hbar\omega_n\beta S^z(a^+ + a) - \mu(S^+E_{\mathrm{pu}}e^{-i\omega_{\mathrm{pu}}t} + S^-E^*_{\mathrm{pu}}e^{i\omega_{\mathrm{pu}}t}) - \mu(S^+E_{\mathrm{pr}}e^{-i\omega_{\mathrm{pr}}t} + S^-E^*_{\mathrm{pr}}e^{i\omega_{\mathrm{pr}}t}). \quad (1)$$

In a rotating frame at the pump field frequency $\omega_{\mathrm{pu}}$, the total Hamiltonian of the system reads as follows:

$$H = \hbar\Delta_{\mathrm{pu}}S^z + \hbar\omega_n a^+ a + \hbar\omega_n\beta S^z(a^+ + a) - \hbar(\Omega S^+ + \Omega^* S^-) - \mu(S^+E_{\mathrm{pr}}e^{-i\delta t} + S^-E^*_{\mathrm{pr}}e^{i\delta t}), \quad (2)$$

where $\Delta_{\mathrm{pu}}=\omega_{\mathrm{ex}}-\omega_{\mathrm{pu}}$, $\Omega=\mu E_{\mathrm{pu}}/\hbar$ is the Rabi frequency of the pump field, and $\delta=\omega_{\mathrm{pr}}-\omega_{\mathrm{pu}}$ is the detuning of the probe field and the pump field.

According to the Heisenberg equation of motion $i\hbar\, dO/dt=[O,H]$, and the commutation relation $[S^z,S^\pm]=\pm S^\pm$, $[S^+,S^-]=2S^z$, $[a,a^+]=1$. The temporal evolutions of the exciton and nanomechanical resonator in the coupled quantum dot-nanomechanical resonator system are given by setting $N=a^+ + a$. In what follows we ignore the quantum properties of $S^z$, $S^-$, and $N$,[22–24] and then the semiclassical equations read as follows:

$$\frac{dS^z}{dt} = -\Gamma_1(S^z + 1/2) + i\Omega S^+ - i\Omega^* S^- + \frac{i\mu E_{\mathrm{pr}}e^{-i\delta t}}{\hbar}S^+ - \frac{i\mu E^*_{\mathrm{pr}}e^{i\delta t}}{\hbar}S^-, \quad (3)$$

$$\frac{dS^-}{dt} = -(i\Delta_{\mathrm{pu}} + \Gamma_2)S^- - i\omega_n\beta NS^- - 2i\Omega S^z - \frac{2i\mu E_{\mathrm{pr}}e^{-i\delta t}}{\hbar}S^z, \quad (4)$$

$$\frac{d^2N}{dt^2} + \gamma_n\frac{dN}{dt} + \omega_n^2 N = -2\omega_n^2\beta S^z, \quad (5)$$

and we have also introduced the damping terms phenomenologically;[19] so here $\Gamma_1$ is the exciton relaxation rate, $\Gamma_2$ is the exciton dephasing rate, and $\gamma_n$ is the decay rate of the nanomechanical resonator due to the coupling to a reservoir of "background" modes and the other intrinsic processes.[1,10,25] In order to solve Eqs. (3)–(5), we make the ansatz[19] $S^z(t)=S^z_0+S^z_+e^{-i\delta t}+S^z_-e^{i\delta t}$, $S^-(t)=S_0+S_+e^{-i\delta t}+S_-e^{i\delta t}$, $N(t)=N_0+N_+e^{-i\delta t}+N_-e^{i\delta t}$. Upon substituting these equations to Eqs. (3)–(5) and upon working to the lowest order in $E_{\mathrm{pr}}$, but to all orders in $E_{\mathrm{pu}}$, we can obtain $S_+$, which corresponds to the linear optical susceptibility as follows: $\chi^{(1)}(\omega_{\mathrm{pr}})=\mu S_+/E_{\mathrm{pr}}=\mu^2/\Gamma_2\hbar\,\chi(\omega_{\mathrm{pr}})$, where the dimensionless susceptibility is given by

$$\chi(\omega_{\mathrm{pr}}) = \frac{2B(\Omega_R^2 w_0 - AD)(\Omega_R^2 + C) + Ew_0}{AE}, \quad (6)$$

where $A=\Delta_{\mathrm{pu0}}-\omega_{n0}\beta w_0-i-\delta_0$, $B=\Delta_{\mathrm{pu0}}-\omega_{n0}\beta w_0+i+\delta_0$, $C=\Omega_R^2\omega_{n0}\beta\eta w_0/(\Delta_{\mathrm{pu0}}-\omega_{n0}\beta w_0-i)$, $D=\Omega_R^2\omega_{n0}\beta\eta w_0/(\Delta_{\mathrm{pu0}}-\omega_{n0}\beta w_0+i)$, $E=\Omega_R^2[-(\delta_0+2i)AB-2B(C+\Omega_R^2)+2A(D+\Omega_R^2)]$, and $\eta=\omega_{n0}^2/\omega_{n0}^2-\delta_0^2-i\delta_0\gamma_{n0}$, where $w_0=2S^z_0$, $\delta_0=\delta/\Gamma_2$, $\Omega_R=\Omega/\Gamma_2$, $\omega_{n0}=\omega_n/\Gamma_2$, $\gamma_{n0}=\gamma_n/\Gamma_2$, $\Delta_{\mathrm{pu0}}=\Delta_{\mathrm{pu}}/\Gamma_2$, and $\Gamma_1=2\Gamma_2$. The population inversion ($w_0$) of the exciton is determined by the following equation:

$$(w_0+1)[(\Delta_{\mathrm{pu0}}-\omega_{n0}\beta w_0)^2+1]+2\Omega_R^2 w_0 = 0. \quad (7)$$

For illustration of the numerical results, we choose the realistic coupled system of an InAs quantum dot and a GaAs nanomechanical resonator, the relevant coupling strength $\beta=0.06$, $\Gamma_1=0.3$ GHz, $\omega_n=1.2$ GHz, and $Q=3\times10^4$.[10] The dephasing rate of exciton is $\Gamma_2=\Gamma_1/2=0.15$ GHz, and the decay rate of the nanomechanical resonator is $\gamma_n=\omega_n/Q=4.0\times10^{-5}$ GHz.

Figure 2(a) shows the absorption spectrum of a probe field as a function of pump-probe detuning. In the middle of the figure, we see that three features appear in the probe absorption spectrum as those in atomic two-level systems.[19]

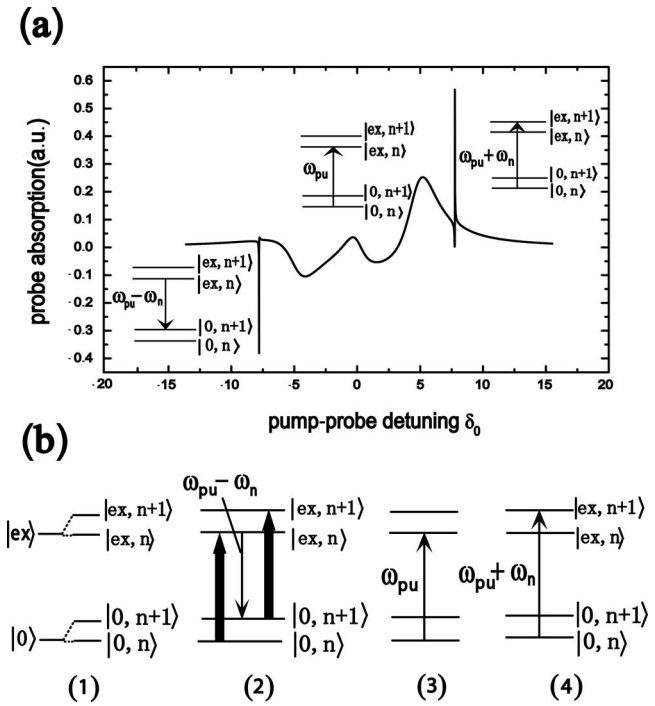


FIG. 2. (a) The absorption spectrum of a probe field in the presence of a strong pump field for the case $\Omega_R^2=6$, $\omega_{n0}=8$, $\Delta_{\mathrm{pu0}}=1$, $\gamma_{n0}=3.0\times10^{-4}$, and $\beta=0.06$. (b) The new features in the spectrum shown in (a) are identified by the corresponding transition between the dressed states of exciton.

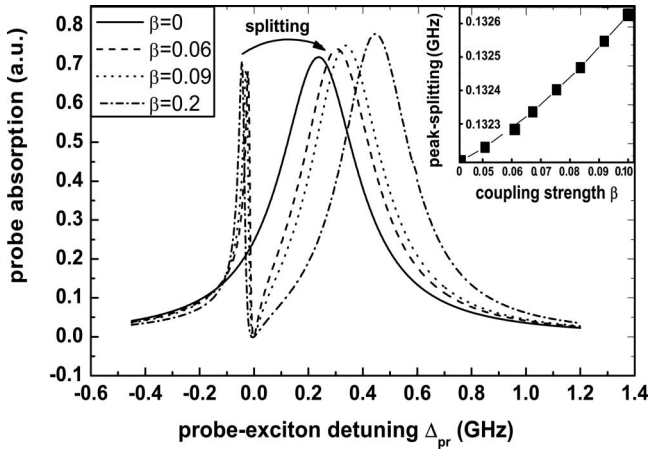


FIG. 3. The absorption spectrum of a probe field as a function of the detuning between a probe field and exciton for three different frequencies of nanomechanical resonator. The other parameters used are $\Omega^2=0.16$ $(\mathrm{GHz})^2$, $\Delta_{\mathrm{pu}}=0$, $\gamma_n=4.0\times10^{-5}$ GHz, and $\beta=0.06$.

However, the new features that are different from those in atomic systems without nanomechanical resonator also appear in the both sides of the spectrum. In analogy with atomic two-level systems in quantum optics, we call this effect as mechanically induced coherent population oscillation. Figure 2(b) gives the origin of these new features. The leftmost (1) of Fig. 2(b) shows the dressed states of exciton ($|n\rangle$ denotes the number states of the nanomechanical resonator). Part (2) shows the origin of mechanically induced three-photon resonance. Here the electron makes a transition from the lowest dressed level $|0,n\rangle$ to the highest dressed level $|\mathrm{ex},n+1\rangle$ by the simultaneous absorption of two pump photons and emission of a photon at $\omega_{\mathrm{pu}}-\omega_n$. This process can amplify a wave at $\delta_0=-\omega_{n0}=-8$, as indicated by the region of negative absorption in Fig. 2(a). Part (3) in Fig. 2(b) shows the origin of mechanically induced stimulated Rayleigh resonance. The Rayleigh resonance corresponds to a transition from the lowest dressed level $|0,n\rangle$ to the dressed level $|\mathrm{ex},n\rangle$. Each of these transitions is centered on the frequency of the pump laser. The rightmost part (4) corresponds to the usual absorption resonance as modified by the ac Stark effect. Therefore, Fig. 2 provides a simple optical method to measure the frequency of nanomechanical resonator. If we first fix the pump detuning $\Delta_{\mathrm{pu}}$ and scan the probe frequency across the exciton frequency $\omega_{\mathrm{ex}}$, then we can easily obtain the vibrational frequency of the nanomechanical resonator in the probe absorption spectrum.

The probe absorption spectrum as a function of the detuning between a probe field and exciton with or without the coupling between quantum dot and nanomechanical resonator is shown in Fig. 3. The parameters used in this figure are $\Omega^2=0.16$ $(\mathrm{GHz})^2$, $\omega_n=1.2$ GHz, $\Delta_{\mathrm{pu}}=1.2$ GHz, and $\gamma_n=4.0\times10^{-5}$ GHz. It exhibits that the coupling strength can be measured by the peak splitting. There is only an absorption peak (solid line) in the absence of coupling between quantum dot and resonator. If the coupling of quantum dot and resonator turns on, then the absorption spectrum will split into two peaks and have a zero absorption point at $\Delta_{\mathrm{pr}}=0$. This is due to mechanically induced coherent population oscillation which makes a deep hole at $\Delta_{\mathrm{pr}}=0$ in the probe absorption spectrum as the pump-probe detuning $\delta$ is equal to the frequency $\omega_n$ of the nanomechanical resonator. The separation between the two peaks shows a strong dependence on the interaction between quantum dot and nanomechanical resonator. We plot the splitting of two peaks as a function of the coupling strength as shown in the inset of Fig. 3. It is obvious that the splitting depends linearly on the coupling strength and goes to 0 in the absence of the coupling. This peak splitting is very similar to the Rabi splitting of two-level systems in quantum optics. The plot provides an effective and accurate method to measure the coupling strength of quantum dot and nanomechanical resonator. Therefore, we can obtain the coupling strength by only simply measuring the distance of two peaks in the probe absorption spectrum.

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